KNOWLEDGE

IN S.T.E.M

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DEFINITIONS

- Axiom - Rules that we take true for granted i.e. $a + b = b + a$ (Peano Axioms). These seem obviously true, but they do not have a proof. We take for granted that they are true.

- Fermat's Last Theorem -

$$\text{There exists no } a, b, c, n \text{ that satisfy the equation}$$

$$a^n + b^n = c^n$$

$$\text{where } a, b, c, n \in \mathbb{Z}^+ \text{ and } n > 2.$$

REAL LIFE SITUATION


QUANTIZED COLUMNS

Why the Proof of Fermat's Last Theorem Doesn't Need to Be Enhanced

24

Decades after the landmark proof of Fermat's Last Theorem, ideas abound for how to make it even more reliable. But such efforts reflect a deep misunderstanding of what makes the proof so important.

Our real life situation focuses upon the enforced strict rules of Mathematics when creating knowledge - i.e. proofs. Specifically, the article mentions that one of the most recent theorems, Fermat's Last Theorem does not require any enhancement or modification, despite not following these 'rules'. Therefore, when producing knowledge we asked ourselves the question: what is the 'correct' way?

INTO THE ARTICLE

- The article we have chosen discusses how the proof of one of the longest unsolved mathematical problems came to be proven in the non-traditional method by Andrew Wiles.
- When two formalist logicians said that they wanted to enhance the proof of Andrew Wiles using computer based confirmations, they got no interest from the number theorists, who have already added FLT to their tool-kit.
- The article talks about how this non-traditional proof is still knowledge that can be utilised, and does not need to be enhanced by taking a traditional viewpoint.

NOTABLE QUOTES

"THE METHODS INTRODUCED BY WILES AND TAYLOR ARE NOW PART OF THE TOOLKIT OF NUMBER THEORISTS"

$$a^n + b^n = c^n$$

"FOR FORMALISTS, A MATHEMATICAL PROOF IS A LIST OF STATEMENTS THAT MEET STRICT REQUIREMENTS"

NOTABLE QUOTES

$$a^n + b^n = c^n$$

"A LOGICAL ANALYSIS OF WILES' PROOF POINTS TO MANY STEPS THAT APPEAR TO DISREGARD ZFC, AND THIS IS POTENTIALLY SCANDALOUS: WHEN MATHEMATICIANS MAKE UP RULES WITHOUT CHECKING THEIR CONSTITUTIONALITY, HOW CAN THEY KNOW THAT EVERYONE MEANS THE SAME THING?"

"IF ASKED TO REPRODUCE THE PROOF AS A SEQUENCE OF LOGICAL DEDUCTIONS, THEY WOULD UNDOUBTEDLY HAVE COME UP WITH 10 DIFFERENT VERSIONS."

KNOWLEDGE QUESTIONS, AOKS AND WOKS

- To what extent are some of our traditional methods of producing knowledge effective?
- To what degree can we approach knowledge from different methods to reach the same conclusion?

AREAS OF KNOWLEDGE

- Mathematics
- Natural Sciences
- Human Sciences

WAYS OF KNOWING

- Reason
- Intuition
- Logic
- Faith

TO WHAT EXTENT ARE SOME OF OUR TRADITIONAL METHODS OF PRODUCING KNOWLEDGE EFFECTIVE?

AXIOMS IN MATHEMATICS

- Axioms in mathematics set the fundamental rules.

- Some axioms can be problematic. For example, there is a system of axioms called "Zermelo-Fraenkel Plus Choice" (ZFC) in Set theory, and the Choice axiom is very problematic.

"For example, you can use Axioms of Choice to prove that it is possible to cut a sphere into five pieces and reassemble them to make *two* spheres, each identical in size to the initial sphere. This is only a theoretical concept – the required cuts are fractal (an infinite pattern), which means they can't actually exist in real life, and some of the pieces are "non-measurable" which means that they don't have a volume defined. But the fact that the Axiom of Choice can be used to construct these impossible cuts is quite concerning."

- And thus, how can we have an axiom that creates the impossible in reality, but not in theory and check its validity?

FORMALISM

- Formalism is an idea developed developed by David Hilbert, and it is an attempt to standardise and keep mathematics consistent amongst people.
- This would effectively allow us to communicate our mathematical knowledge across people.
- This is a very standard way of creating knowledge within our society. We love to keep things consistent to make it easy for us. It is a traditional way of creating knowledge.

Figure 1: David Hilbert

THE CREATION OF AXIOMS TO CREATE KNOWLEDGE

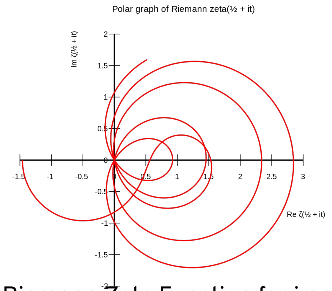


Figure 2: Riemann Zeta Function for imaginaries

- In order to create knowledge in mathematics, we need axioms. However, what if the knowledge is unreachable using our current axioms?

- One famous conjecture that is argued not being able to be proven using our axioms is the Goldbach's conjecture (every even integer greater than 2 is the sum of 2 primes).

- In fact, the idea that not all theorems can be proven using an axiomatic system has been proven using our axioms. This is called the Gödel and the Incompleteness Theorem.

- One mathematical example of this is called the 'Continuum Hypothesis', more specifically, "The continuum hypothesis could be true or false, and it is impossible to prove it either way. It basically means that you can decide for yourself whether you *want* it to be true or not."

- Today, we come in an agreement that it is okay to create your own axioms, and that there exists infinitely many such undiscovered/produced axioms. However, how can we keep the axiomatic system consistent then?

THE RELATIONSHIP WITH WILES' PROOF

- However, some of the latest proofs do not follow the formalist traditional method of creating knowledge (i.e. be limited to certain axioms).

"A logical analysis of Wiles' proof points to many steps that appear to disregard ZFC, and this is potentially scandalous: When mathematicians make up rules without checking their constitutionality, how can they know that everyone means the same thing?"

- As a result, some of the newly produced knowledges such as Wiles' proof arguably needs 'standardisation' to make sure that everyone is able to understand what is happening. If all knowledge is unique and no knowledge corresponds to today's 'widely' accepted knowledge, how can we make sure that such knowledge is consistent?

IS THE KNOWLEDGE "VALID"?

- Indeed, it is possible to argue that through the newly defined axioms within the Fermat's Last Theorem proof is perfectly valid. However, how would we prove the validity of axioms?

- If axioms are indeed systems of knowledge that we take for granted, then arguably it is a system whose basis is made out of:

  Faith - i.e. in hope that such statements are indeed true to some extent so we can apply them to real life to create knowledge
  Logic - i.e. does the axiom we create have correct reasoning that makes sense in our mind
  Reason - i.e. would the existence of such an axiom be consistent within our current axioms
  Intuition - i.e. what we believe is correct and what our guts tell us what has to be true even if we do not have a formal proof to explain a certain phenomena

- Therefore, due to the definition of validity we have set up today, "(of an argument or point) having a sound basis in logic or fact; reasonable or cogent." - as axioms are based off of logic as well as reason, they're valid.

Show that $n^5 - n$ is divisibly by 30 for all $n \in \mathbb{Z}^+$

Factorising

$$\implies n(n^4 - 1)$$
$$\implies n(n^2 + 1)(n^2 - 1)$$
$$\implies n(n + 1)(n - 1)(n^2 + 1)$$

The product of 3 consecutive integers is always divisible by 3.
The product of 2 consecutive integers is always divisible by 2.
Consider

$$\implies n(n + 1)(n - 1)(n + 2)(n - 2) + 5$$

The product of 5 consecutive integers is always divisible by 5.
$\therefore$ 30 divides into $n^5 - n$ for all $n \in \mathbb{Z}^+$

By Fermat's Little Theorem,

$$n^2 \equiv n \bmod 2$$

Raising everything to the power of 3 and dividing by n

$$n^5 \equiv n^2 \bmod 2$$

From our first equation, then

$$n^5 \equiv n \bmod 2$$

$\therefore$ 2 divides into $n^5 - n$
By Fermat's Little Theorem,

$$n^3 \equiv n \bmod 3$$

Squaring everything and dividing by n

$$n^5 \equiv n \bmod 3$$

$\therefore$ 3 divides into $n^5 - n$
By Fermat's Little Theorem,

$$n^5 \equiv n \bmod 5$$

$\therefore$ 5 divides into $n^5 - n$
$\therefore$ 30 divides into $n^5 - n$

And therefore by the definition of "reliability" "The quality of being trustworthy or of performing consistently well." mathematics as a system of knowledge we have created is also reliable.

BUT IT DOESN'T MEAN IT IS NECESSARILY THE TRUTH

- Whilst the idea of validity and reliability might re-assure the idea that it is a system that can be close to achieve truth, this may be wrong.

- Mathematics is a human language, not the universe's language. It is a language that we have created when creating axioms, and may not reflect the world.

- The real 'truth' may be unreachable, because it is not systematic. There are no patterns. There is only chaos. The world is simply is not as nice as we think it is.

- Whilst the truth may not be 100% reachable, our current system of mathematics has allowed us to create vast knowledge and understanding of what we can indeed achieve. Errors are bound to happen due to the idea that the 'truth' not being reachable, but we can be close in recreating the 'truth' in our own made up world of mathematics, and taking in account of this by adding 'uncertainty' factor.

POSITIVISM

- Positivism is a system that is based on proving a theory through mathematical data, natural sciences phenomena and evidence. They believe that the basis of truth is hidden within numbers, and that numbers govern the world.

  In maths, we can do this by using counterexamples:

- E.g. The hypothesis that all prime numbers are odd can be disproven by 2, and the number 2 is the only number that is even and prime.

Figure 4:
Auguste Comte

FALSIFICATION

- Karl Popper believed that for a theory to be scientific, it could be proven false.

- According to Popper, science should be about disproving theories as opposed to continuously support them with evidence and such, which is what most of our society is based on right now.

- Popper believes scientific knowledge is still provisional and the best we can do right now.

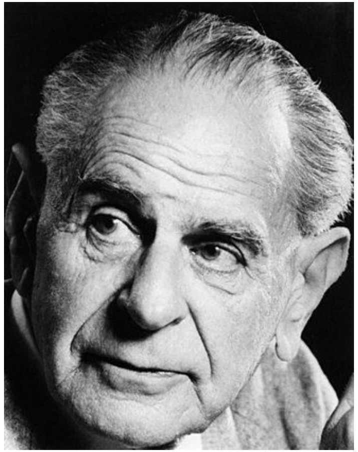


Figure 3:
Karl Popper

POSITIVISM IN HUMAN SCIENCES

- Positivism is very frequently used in human sciences in order to create statistical data and as a result produce knowledge.

- For example, sociology explores upon the idea of human behaviour within society. Recall the idea that with the Chaos Theory, not everything has to fall in predictable motion because in reality, it could be much more complex.

- Because the human behaviour is a very mysterious phenomenon, we have multiple methods in an attempt to understand it and create knowledge - and this is in fact not strictly limited to the use of mathematics as well as numbers.

Consider emotion:

- How can we express anger with numbers like we depict the movement of a particle?

INTERPRETIVISM

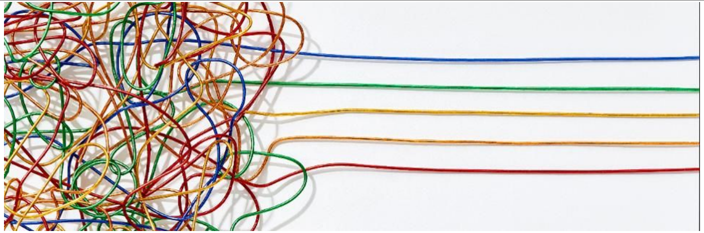


Figure 5: The visualisation of knowledge interpretations

- Interpretivists acknowledge that some things are just too incredibly complex to label them with numbers, therefore the system of knowledge must be taken as a whole and considered that way.

- Why did we take the positivist approach in interpreting natural sciences but consider interpretivism within human sciences?

- Human behaviour is more complex to understand and gather knowledge of than our surroundings. Yet we still use numbers and a positivist approach in areas such as economics, which can simplify human behaviour.

- Natural sciences for example, arguably makes use of both interpretivism and positivism along with their qualitative and quantitative methodologies. Biology and chemistry, for example, are more qualitative whilst physics is more quantitative.

TO WHAT DEGREE CAN WE APPROACH KNOWLEDGE FROM DIFFERENT METHODOLOGIES TO REACH THE SAME CONCLUSION?

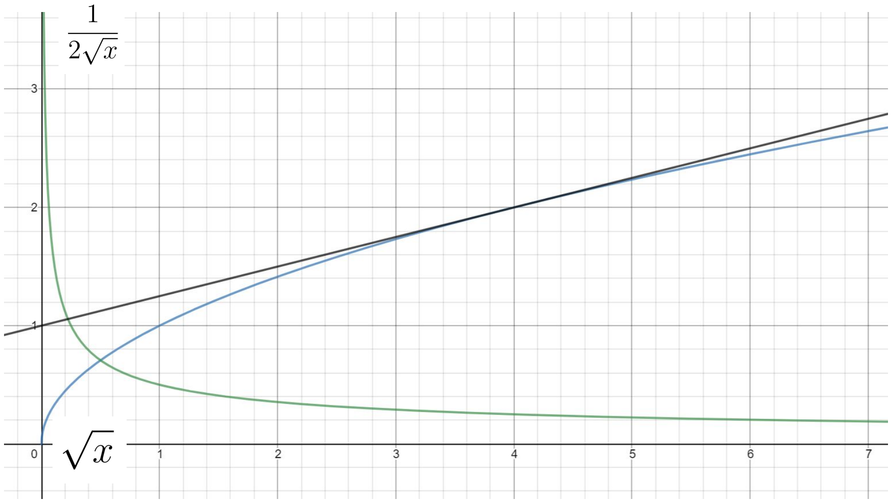
PERSONAL EXPERIENCE -

DISTANCE TO VELOCITY

- For a graph that measures displacement, we can take the velocity of a certain point on the graph through two different methodologies: using calculus or taking the tangent of that point

Gradient Formula: $$\frac{y_2 - y_1}{x_2 - x_1} = m$$

Substitute x=4 into $$\frac{1}{2\sqrt{x}}$$



ARE THEY THE SAME?

Yes.

For someone who doesn't know about what calculus they would think that these are two different approaches to reach the same conclusion, but differentiation itself is a function that follows the principle of

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

The crucial difference between them is that differentiation makes the difference between y2 and y1 so small that it creates a function of itself that can be done to find the gradient of any point on the curve. That is what

$$\frac{dy}{dx}$$

is. It is the difference in y / the difference in x, the same thing as just getting two points from the tangent.

BUT IT ISN'T THE SAME METHODOLOGY

- In a physics or maths exam a student could potentially do both, but one student would get method marks whilst the other student would not (as frequently happened in our past classes). This is because differentiation is a more developed version of using the tangent. However, when a student has the graph but not the function, they may have to draw a tangent and find points on the tangent to get the gradient. They follow the same principles but they are not the exact same.

- They are different tools that follow the same principles to reach the same conclusion, yet they use different information. You can't differentiate without the function of the graph, and you can't use the tangent without values on the x and y axis.

THE PROOF OF ATOMS

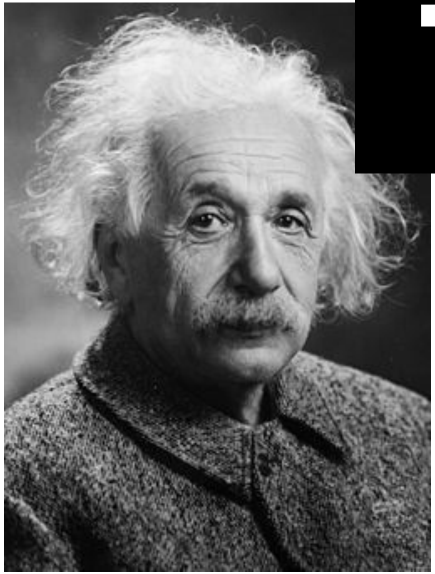


Figure 6: Albert Einstein

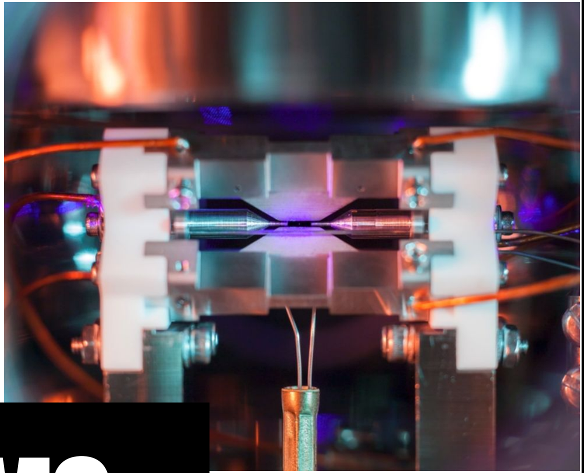


Figure 7: The first picture of the atom

BROWNIAN MOTION

- The motion of particles suspended in air or liquid darting around is named Brownian Motion.

- Though Einstein wasn't the first person to describe the Brownian motion mathematically, he was the one who concluded that the mathematical description of Brownian motion was evidence for the existence of atoms.

- This is a scientific noumenon for the atom. as even though we can see the dust particle darting around we can't see the atoms that are making the dust particle dart around.

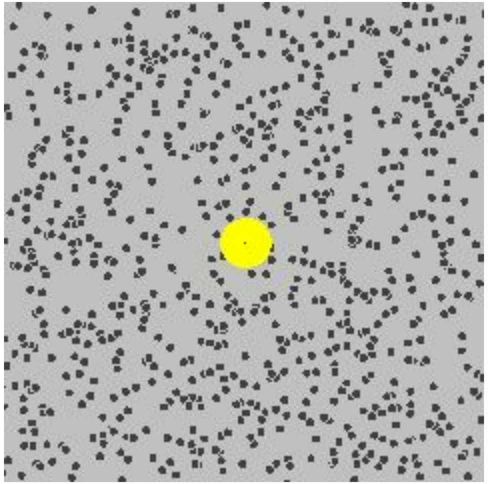


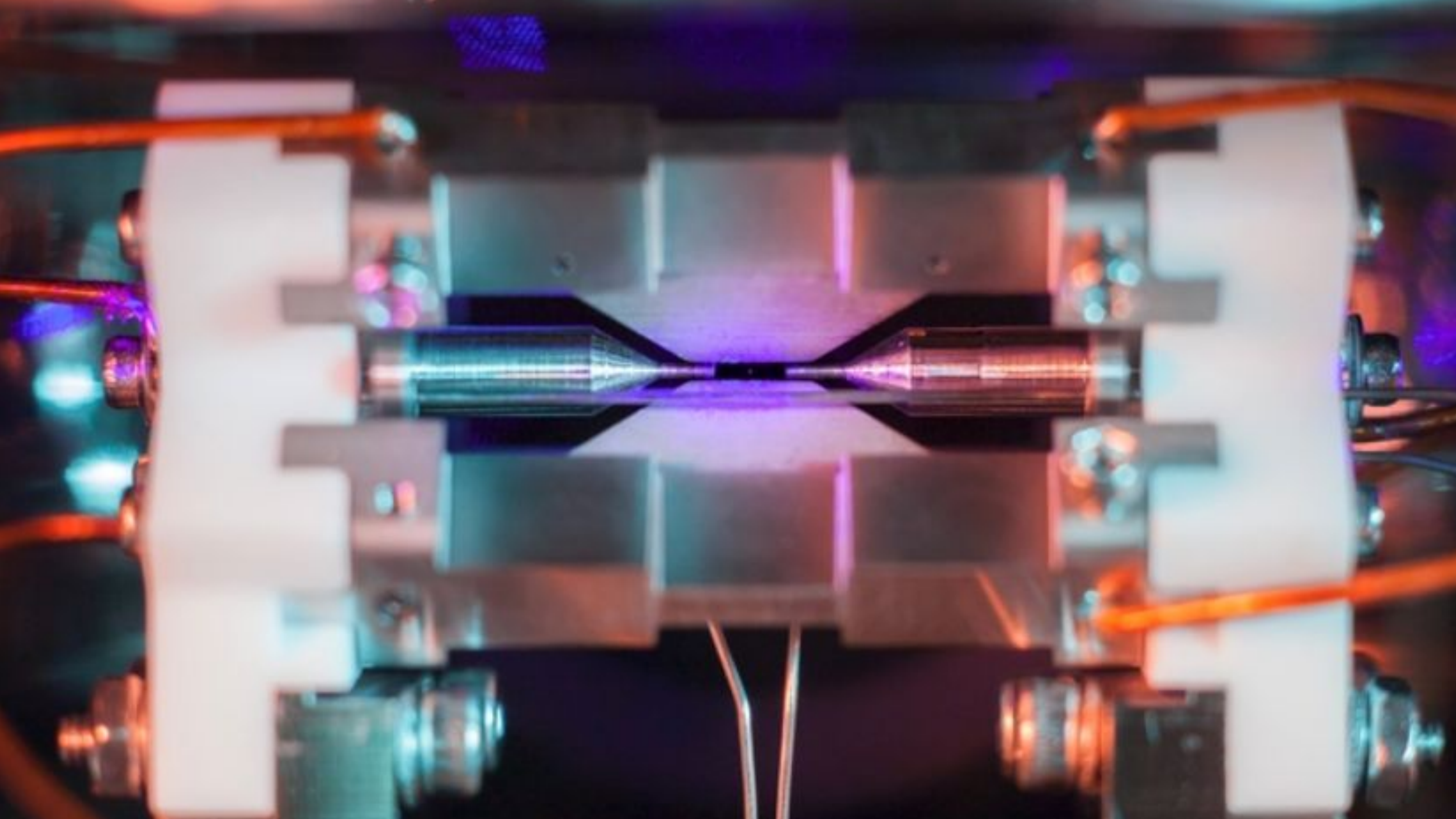
Figure 8: Brownian Motion

THE FIRST PHOTO OF THE ATOM

The first photo of the atom of Strontium was taken by David Nadlinger from the University of Oxford.

The atom was suspended in an ion trap. It was illuminated by a laser of a precise blue-violet colour that the atom absorbed and re-emitted, quickly enough for a normal camera to capture it in a long exposure photograph. This was done in a ultra-high vacuum chamber, which houses the ion trap.

This is a scientific phenomena as we can see the atom, thus meaning it a directly observable fact.



ARE THESE PROOFS?

- They are a noumenon and phenomenon respectively, but how can we tell that this very thing we observed is truly a fact?
- When making theories we always support them. However, according to Popper's idea of falsification, we can't scientifically take for granted something that can't be proven false.
- As Popper said, science is still provisional and that we can't exactly prove something. What is to say that brownian motion isn't caused by yet another subatomic particle, or some type of force such as gravity that we still have not discovered?
- Through the positivist viewpoints we have a good idea on what is correct and what is not, as it is crucial for the development of humankind.

CONCLUSION

- The axiomatic system within mathematics to produce knowledge is limited, and has been proven using these axioms. Not limiting ourselves to the traditional axioms may be required in order to produce knowledge closer to the truth.

- The system of rules within mathematics that have created may be too ‘generous’ or too ‘ideal’ in comparison to the real world and truth.

- It is possible that we shouldn’t limit our knowledge production to favouring a certain method, that is, either positivism, interpretivism, formalism etc.

CONCLUSION

- In natural sciences though our understanding is pretty much set in stone, we still can not prove anything 100%, though we can continue providing evidence for it.

- Moreover, it may be possible that the reason we are able to provide proof in mathematics is because the world of math is “ideal”. The world of natural sciences and human sciences on the other hand, is an attempt to understand the ‘truth’ of the universe which is not ideal

- We can reach the truth as close as we can if we attempt to utilise everything

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THANKS

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